

RECENT AERODYNAMIC CONTRIBUTIONS  
TO PROBLEMS OF BIRD FLIGHT

D. Hummel

Institut für Strömungsmechanik  
Technische Universität Braunschweig  
Federal Republic of Germany

Abstract

A survey on recent aerodynamic contributions to phenomena of bird flight is given. Three problems are studied in some detail by theory and partly by experiment, namely

- (i) the formation flight of birds, which leads to a reduction of flight power demand.
- (ii) the slotted wing-tips of soaring birds, by which static stability is increased and induced drag is reduced.
- (iii) the flapping flight of birds, for which spanwise distributions of vertical force and thrust have been calculated.

The discussion of the results indicates that the application of the methods of aerodynamics leads to a better understanding of flight in nature.

1. Introduction

In the early days of aeronautics the flight of birds has stimulated aeronautical engineers very much. The principle of human flight has been learnt from nature and at that time almost all pioneers in this field such as for instance O. Lilienthal [1] have studied the flight of birds. After the first successful human flights at the beginning of this century the aeronautical sciences on the one hand have developed as a part of physics and separate from biology. The flight of birds was no longer important for the development of the new scientific discipline, even in details. The biological sciences on the other hand continued to investigate the flight of birds mainly by observation and description such as for instance by K. Lorenz [2] and later by analysing slow motion films; see G. Rüppell [3] and also W. Nachtigall [4]. The interpretation of the observations turned out to be very difficult for biologists and aerodynamicists too.

The problems of bird flight can only be solved using the experience and the methods of the well established aeronau-

tical sciences. In addition to this it is necessary to analyse the geometry and the motion of birds in flight quantitatively in order to get precise formulations of the problems which are the basis for the application of the methods of aerodynamics. In the last years remarkable progress has been achieved by biologists in the quantitative determination of positions and motions of birds in flight. The solution of some corresponding aerodynamic problems is now possible and following here some selected examples of this kind are presented.

2. Notations

2.1 Symbols

$\mu = b^2/S$	aspect ratio of wing $\mu$
$b$	reference span (Section 3), span
$b_\mu$	span of wing $\mu$
$B_\mu = b_\mu/b$	span ratio
$b'$	reduced span, equ. (2)
$c$	chord
$c_L = L/qS$	lift coefficient
$c_D = D/qS$	drag coefficient
$c_h$	vertical force coefficient, Fig. 16
$c_t$	thrust coefficient, Fig. 16
$c_{Di}$	induced drag coefficient
$c_{Dp}$	profile drag coefficient
$e$	local relative power reduction, equ. (10)
$E$	total relative power reduction, equ. (14)
$f$	geometric function, equ. (4)
$k$	wing sections, Figs. 14, 15, 16
$n$	number of wings in formations (Section 3) or number of winglets (Section 4)
$N$	flight power demand
$q = \rho V^2/2$	dynamic pressure
$s = b/2$	half-span

S	wing area
V	flight speed
w	upwash velocity, equ. (4)
$\bar{w}$	mean value of w, equ. (1)
W	weight
x,y	rectangular coordinates
$y_0$	unsplitted part of one half wing, Fig. 10
$\delta$	stagger angle
$\epsilon$	twisting angle
$\eta = y/s$	dimensionless spanwise coordinate
$\eta_0 = y_0/s$	splitting ratio
$\Delta\eta = \Delta y/b$	dimensionless spanwise distance
$\Gamma$	circulation, equ. (3)

## 2.2 Subscripts

ell	elliptical circulation-distribution
l	leading
m	minimum
t	trailing (Section 4), thrust (Section 5)
o	single flight
$\mu$	wing under consideration
v	inducing wing.

## 3. Formation flight of birds

### 3.1 Background

It is well known that a number of migrating birds such as for instance Swans, Geese and Cranes fly in regular V-shaped formations as shown in Fig. 1. The birds are usually ordered in swept lines and they keep so small spanwise distances that the wing-tips of two adjoining birds lie about one behind the other.

In the biological sciences different opinions exist concerning the mechanism of formation flight. E. Stresemann [5], M. Stolpe and K. Zimmer [6] and many others suppose that in such formations a power reduction occurs for each individual, whereas L. Franzisket [7] and others deny energetic benefits and assume that regular formations of birds occur only for reasons of good optical contact. In the aeronautical sciences this problem has also been considered with respect to the formation flight of airplanes. C. Wieselsberger [8] was the first to give the correct explanation of the power reduction in such formations. After the development of wing theory H. Schlichting [9] performed some calculations of the drag reduction in symmetrical V-shaped flight formations. Later P.B.S. Lissaman and C.A. Shollenberger [10] investigated optimum formation shapes and D. Hummel [11,12] extended the method of H. Schlichting [9] to arbitrarily shaped formations of wings having different span, weight and aspect ratio. Following here the formation flight will be analysed in some detail.

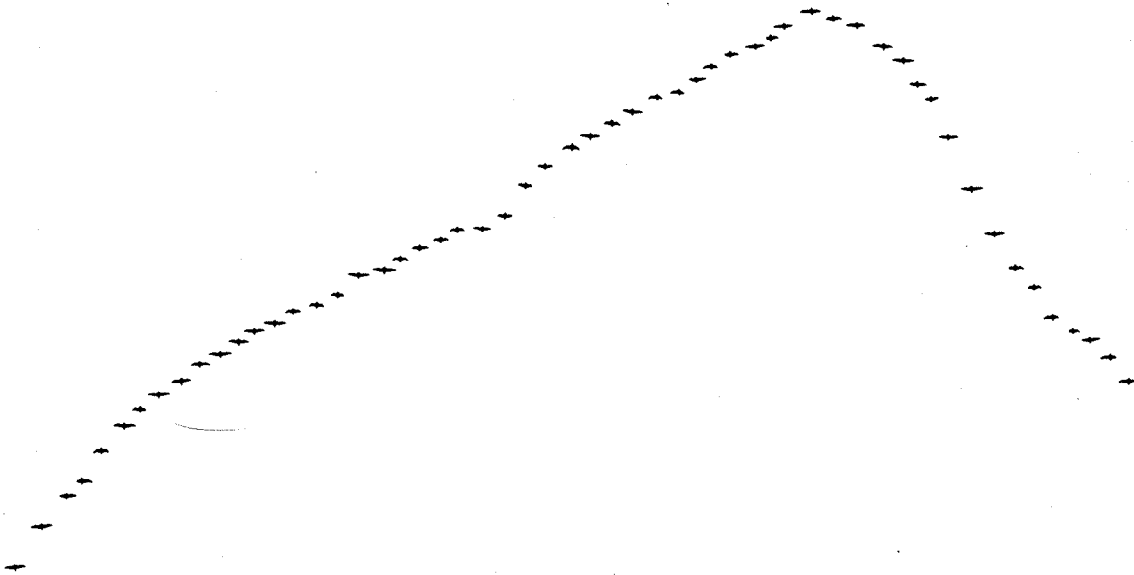


Fig.1: V-shaped flight formation of migrating geese (*Branta bernicla*).

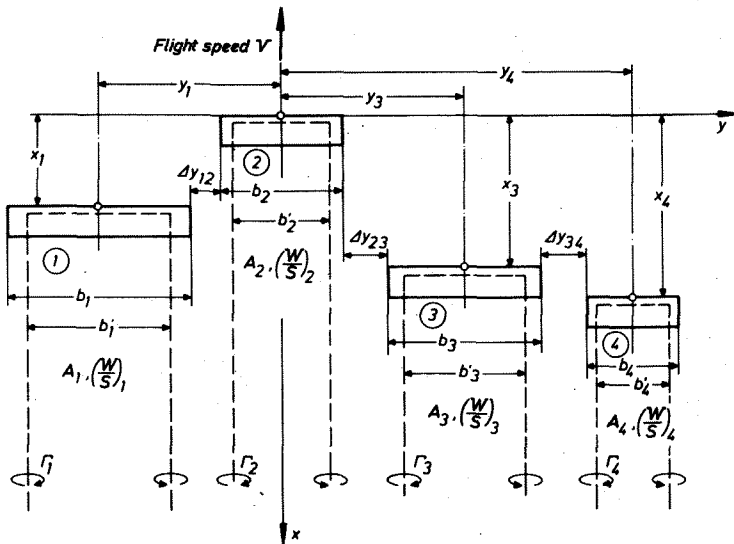
### 3.2 Theoretical analysis

The investigations, following here, are concerned with arbitrarily shaped flight formations of different wings in horizontal flight according to Fig. 2. It is assumed that the shape of the formation remains constant with respect to time. This means that all wings fly at the same speed  $V$ . The corresponding dynamic pressure is  $q$ . It is further assumed that the speed of the formation is the same as that of each wing in single flight which is used as basis for comparisons. The different size of the wings is described by their span  $b_\mu$  ( $\mu = 1, 2, \dots, n$ ). If all wing spans are related to a reference span  $b$ , the corresponding dimensionless parameter is the span-ratio  $B_\mu = b_\mu/b$ . The planform shape of the wings  $\mu$  is characterized by the aspect ratio  $A_\mu = b_\mu^2/S_\mu$ . For given values of  $A_\mu$  and  $B_\mu$  the wing area is  $S_\mu/b_\mu^2 = B_\mu^2/A_\mu$ . In the various positions  $\mu$  of the formation the wing loading  $W_\mu/S_\mu$  may be different. According to the balance of the vertical forces the corresponding dimensionless parameter is the lift coefficient  $c_{L\mu} = (W_\mu/S_\mu)/q$  of each wing  $\mu$ .

In horizontal flight of a single wing  $\mu$  (index 0) the lift  $L_{0\mu}$  equals the weight  $W_\mu$  and the power demand  $N_{0\mu}$  is  $N_{0\mu} = D_{0\mu} V$ . The drag consists of the friction drag and of the induced drag whereas the drag due to the flapping motion of the wings can be neglected [10].

The aerodynamic principle in flight formations is the fact that each wing  $\mu$  of the formation flies in an upwash field generated by all other wings  $\nu$  of the formation. Due to the upwash  $\bar{w}_\mu$  at each wing  $\mu$  the flow direction and thus the direction of the lift ( $L_\mu = L_{0\mu}$ ) is turned at an angle

$$\varphi_\mu = \frac{\bar{w}_\mu}{V} \quad (1)$$



A simple analysis leads to an estimate of the upwash  $\bar{w}$ : Each inducing wing  $\nu$  can be replaced by  $\mu$  a single horseshoe-vortex of span

$$b'_\nu = \frac{\pi}{4} b_\nu \quad (2)$$

and circulation

$$\Gamma_\nu = \frac{L_\nu}{qVb'_\nu} = \frac{2}{\pi} \frac{c_{L\nu}}{A_\nu} V b_\nu \quad (3)$$

according to Fig. 2. The induced upwash  $w_{\mu\nu}$  at the wing  $\mu$  generated by this  $\mu\nu$  vortex system of the wing  $\nu$  is

$$\frac{w_{\mu\nu}(y_\mu)}{V} = \frac{c_{L\nu}}{\pi A_\nu} f_{\mu\nu}(y_\mu) \quad (4)$$

where  $f_{\mu\nu}$  is a geometric function given by Biot-Savart's law. The upwash according to equ. (4) is a very good approximation for the values produced by a plane vortex sheet [9] having an elliptic spanwise distribution of circulation. Since the upwash  $w_{\mu\nu}$  is a function of the spanwise coordinate  $y_\mu$  an integral mean value is calculated according to

$$\frac{\bar{w}_{\mu\nu}}{V} = \frac{c_{L\nu}}{\pi A_\nu} \bar{F}_{\mu\nu} \quad (5)$$

with

$$\bar{F}_{\mu\nu} = \frac{1}{b'_\mu} \int_{-b'_\mu/2}^{+b'_\mu/2} f_{\mu\nu}(y_\mu) dy_\mu \quad (6)$$

The upwash  $\bar{w}_\mu$  at the wing  $\mu$  generated by all other  $\mu$  wings  $\nu$  of the formation is

$$\frac{\bar{w}_\mu}{V} = \sum_{\nu=1}^n \frac{c_{L\nu}}{\pi A_\nu} \bar{F}_{\mu\nu} \quad (7)$$

Fig. 2: Arrangement of wings in an arbitrarily shaped flight formation (example  $n = 4$ ).

The drag reduction at the wing  $\mu$  due to the upwash angle  $\varphi_\mu$  according to equ. (1) is

$$\Delta D_\mu = L_\mu \frac{\bar{w}_\mu}{V} \quad (8)$$

and the corresponding reduction in flight power demand is

$$\Delta N_\mu = L_\mu \bar{w}_\mu \quad (9)$$

Based on the power demand for the same wing in single flight at the same speed  $V$ , the relative flight power reduction of the wing  $\mu$  is

$$e_\mu = \frac{\Delta N_\mu}{N_{0\mu}} = \frac{L_{0\mu}}{D_{0\mu}} \frac{\bar{w}_\mu}{V} \quad (10)$$

Using equ. (7) and after some rearrangement equ. (10) can be written as

$$e_\mu = \left( \frac{c_{Di,ell}}{c_D} \right)_{\mu 0} \sum_{\nu=1}^n \frac{A_\mu (W/S)_\nu}{A_\nu (W/S)_\mu} \bar{F}_{\mu\nu} \quad (11)$$

The relative power reduction of the wing  $\mu$  under consideration is proportional to the ratio of induced drag (for elliptic distribution of circulation,  $c_{Di,ell} = c_L^2 / \pi A$ ) to total drag at the single flight condition. This ratio is 0.5 for maximum range flight and 0.75 for minimum power demand flight. Equ. (11) shows the influence of the different parameters  $A, B$  and  $W/S$ , where  $B$  as well as the geometry of the formation are included in the different values of the geometric function  $\bar{F}_{\mu\nu}$ .

The total reduction of flight power demand of the whole formation of  $n$  wings is

$$\Delta N = \sum_{\mu=1}^n \Delta N_\mu \quad (12)$$

Based on the power demand of all wings in single flight

$$N_0 = \sum_{\mu=1}^n N_{\mu 0} \quad (13)$$

the relative power reduction of the whole formation can be expressed using equ. (10) as

$$E = \frac{\Delta N}{N_0} = \frac{\sum_{\mu=1}^n e_\mu c_{D\mu 0} B_\mu^2 / A_\mu}{\sum_{\mu=1}^n c_{D\mu 0} B_\mu^2 / A_\mu} \quad (14)$$

This relation shows how the various span ratios  $B$  and the different aspect ratios  $A$  enter anew. The different wing loadings  $W/S$  determine the lift coefficients  $c_{L\mu 0}$  which are connected with the drag coefficients  $c_{D\mu 0}$  through the relation  $c_{L\mu 0}^2 = c_{D\mu 0}^2$ . The calculations have been carried out for quadratic polars

$$c_{D\mu} = c_{Dp\mu} + \frac{c_{L\mu}^2}{\pi A_\mu} \quad (15)$$

and the special value of the profile drag coefficient  $c_{Dp\mu} = 0.03$ .

The present simple analysis has the advantage of showing clearly the influence of certain parameters. The results have been checked by some calculations according to lifting surface theory after E. Truckenbrodt [13] for  $n = 2$  wings. It turns out that for moderate and large spanwise distances of the wings, the results are nearly the same. For very low spanwise distances the present analysis leads to slightly lower values of the power reduction than the more exact solution.

### 3.3 Results and discussion

Example calculations have been carried out for various flight formations. Some results are presented here. Further details may be taken from D. Hummel [11,12].

#### 3.3.1 Homogenous flight formations

In the case of equal wings the problem reduces considerably. The aspect ratio  $A_\mu$  has not to be specified and the span  $B_\mu$  ratio is  $B = 1$ . The only significant parameter is  $\mu$  the drag ratio according to equ. (11) which has been chosen as

$$(c_{Di,ell}/c_D)_{\mu 0} = 0.5$$

for the maximum range condition. Following here, some typical results are presented and discussed.

Fig. 3 shows the total power reduction  $E$  in flight formations of equal wings as a function of spanwise distance  $\Delta\eta = \Delta y/b$

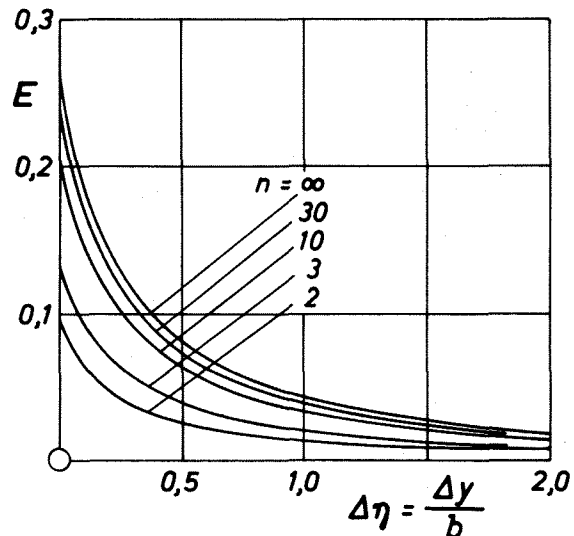


Fig.3: Total power reduction in flight formations of equal wings as a function of spanwise distance and of number of wings,  $(c_{Di,ell}/c_D)_{\mu 0} = 0.5$ .

and of the number of wings  $n$ . It turns out that the power reduction increases considerably with decreasing spanwise distance of the wings. The power reduction increases also with increasing number of wings  $n$ , but there exists a limiting curve for  $n \rightarrow \infty$  which has been given by H. Schlichting [9]. The result of Fig. 3 is independent of the actual shape of the formation. This is the well known displacement-theorem of M.M. Munk [14]. It turns out, that all flight formations having the same spanwise distances between the wings and the same number of wings achieve the same total power reduction.

In flight formations of birds the spanwise distances are narrow. In this condition the total flight power reduction reaches remarkable values, which are the reason for the fact that the behaviour of forming regular formations has been developed in nature by evolution.

The distribution of the power reduction on the wings of the formation de-

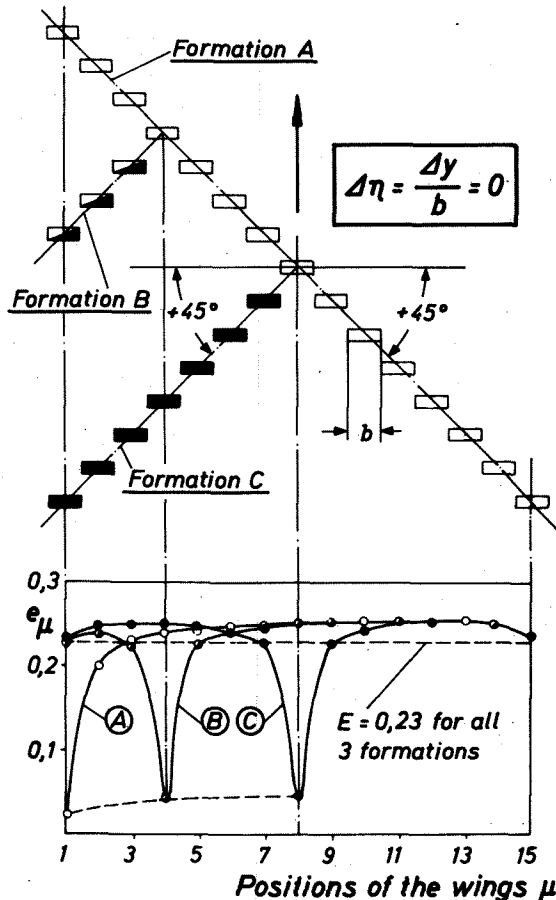
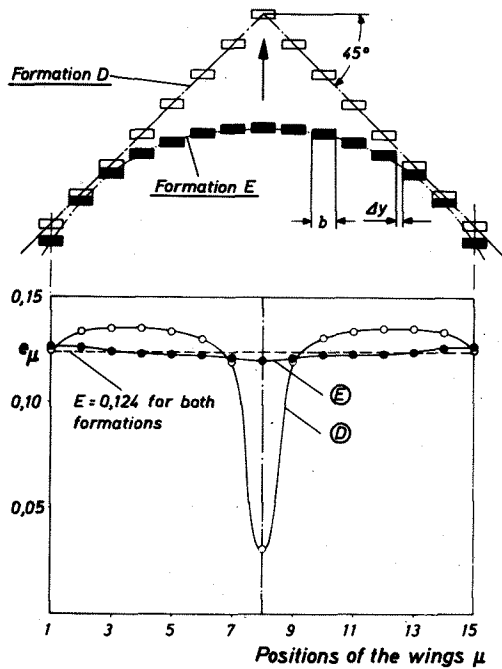


Fig.4: Distribution of flight power reduction in  $45^\circ$  swept V-shaped formations of  $n = 15$  wings at  $\Delta\eta = 0$  and  $(c_{D1,el}/c_D)_0 = 0.5$  for various leading positions  $n_1$ .  
 Formation A: Oblique line,  $n_1 = 1$   
 Formation B: Unsymmetr. formation,  $n_1 = 4$   
 Formation C: Symmetrical Formation,  $n_1 = 8$

pends strongly on the shape of the formation. Fig. 4 shows a typical example for formations of  $n = 15$  equal wings at a spanwise distance  $\Delta\eta = 0$ . The wings are arranged on swept lines and the leading position  $n_1$  is varied from  $n_1 = 0$  (oblique line) over  $n_1 = 4$  (unsymmetrical formation) to  $n_1 = 8$  (symmetrical formation). First of all the total power reduction is the same ( $E = 0,23$ ) for all these formations. In the leading position the power reduction is very low. This is due to the fact that the upwash decreases rapidly in upstream direction. The positions at both sides of the formation are also unfavourable, since no neighbour-wings are present which would produce upwash. The worst position is those of the leading wing of a swept line formation ( $n_1 = 1$ ) since both effects add. In the center of straight lines the power reduction lies over the average value.

V-formations are very often observed in nature though there exist some unfavourable positions. The birds make allowance for this disadvantage in favour of a good optical contact. The sweep angle of the total power reduction of the formation. Nevertheless, birds fly at certain sweep angles which are characteristic for the species. Short-necked birds such as for instance Lapwings (*Vanellus vanellus*) and Black-headed Gulls (*Larus ridibundus*) keep small sweep angles, whereas long-necked birds such as Geese (*Anser spec.*, *Branta spec.*) and Cranes (*Grus grus*) fly at larger sweep angles. An estimate shows that the connecting line between the eye and the wing-tip of the bird, which is important for the optical contact within the formation, coincides with the oblique lines of the formation.

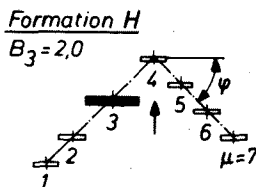
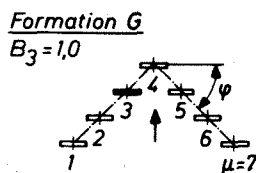
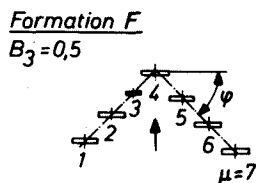
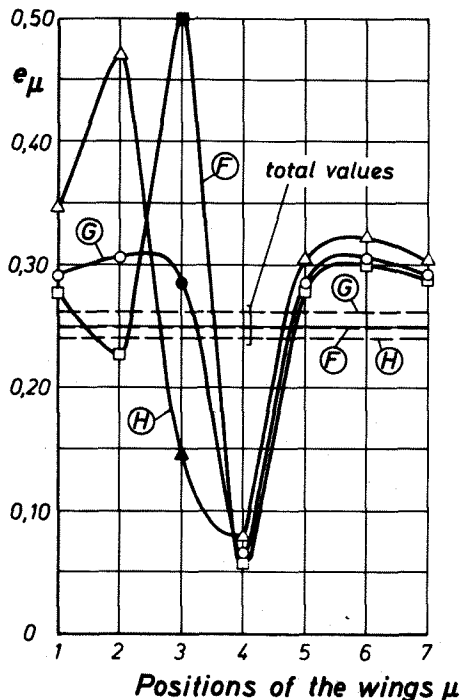
Due to the very low power reduction in the leading position of a formation the corresponding individual should easily become tired. That this is true can be concluded from the fact that changes of the positions in the neighbourhood of the apex of a formation are often observed in flight formations of birds. But for the individuals at the apex of a formation there exists although a possibility to increase their flight power reduction by flying in more rearward located positions. Fig. 5 shows an example of this kind. The positions of the wings have been varied systematically until a nearly constant local power reduction within the formation has been obtained. The  $45^\circ$  swept V-shaped formation having the same number of birds  $n = 15$  at the same spanwise distances  $\Delta\eta = 0,2$  is also drawn for comparison. For constant local power reduction a formation with a considerably rounded apex turns out. Flight formations of this kind are also observed in nature. It must be assumed that in such formations the birds are tired and that they allow optical disadvantages in favour of a proper distribution of flight power reduction.



**Fig.5:** Flight formation of  $n = 15$  wings at  $\Delta\eta = 0,2$  with constant local power reduction,  $(c_{Di,ell}/c_D)_0 = 0,5$ .  
**Formation D:** Symmetr. formation,  $45^\circ$  swept  
**Formation E:** Optimal formation,  $e_\mu \approx \text{const.}$

### 3.3.2 Inhomogenous flight formations

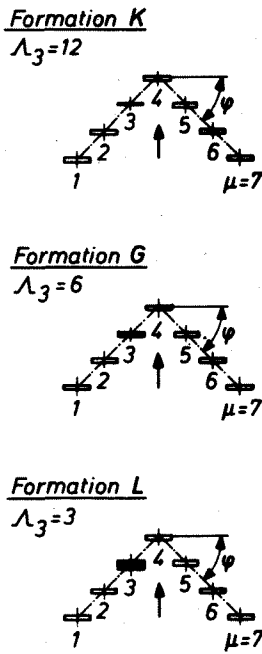
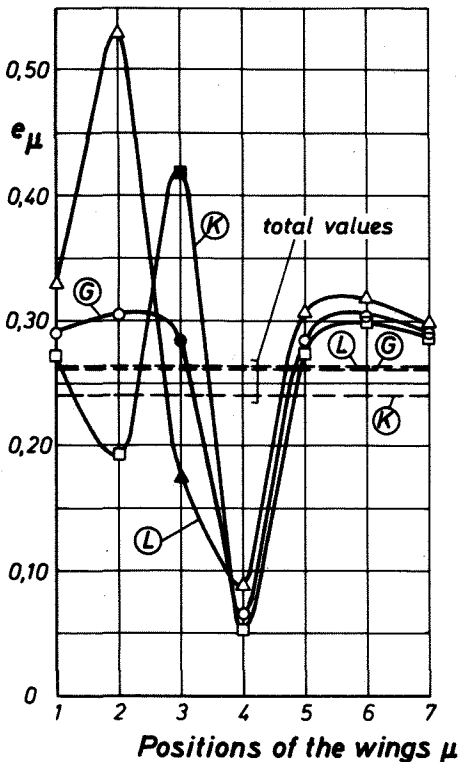
The calculations have been carried out for wings having a quadratic polar according to equ. (15) with a profile drag coefficient of  $c_{D\mu 0} = 0,03$ . Symmetrical V-shaped formations of  $n = 7$  wings have been chosen. The wings were arranged on  $45^\circ$  swept lines.



Different wing spans in a formation are investigated in Fig. 6. The basis of the study is a symmetrical V-shaped formation of equal wings (formation G). In two other formations the wing  $\mu = 3$  is replaced by a wing of half span  $B_3 = 0,5$  (formation F) and of twice span  $B_3 = 2,0$  (formation H). The aspect ratio  $A_3$ , the wing loading  $(W/S)_3$  as well as the spanwise distance between the wing-tips ( $\Delta\eta = 0$ ) remained constant. In the formation F the wings  $\mu \neq 3$  lie closer together than in formation G. Therefore the upwash produced by interference between the wings  $\mu \neq 3$  increases. On the other hand the wing  $\mu = 3$  has in formation F only the half circulation according to equ. (3). The upwash induced by the wing  $\mu = 3$  is therefore considerably reduced. This second effect is predominant. As compared with formation G, the relative power reduction  $e_\mu$  decreases in all positions  $\mu \neq 3$  of  $\mu$  formation F. The reduction of the benefit is extremely large at the downstream neighbour wing  $\mu = 2$ . The smaller wing  $\mu = 3$  is placed in stronger upwash due to the smaller spanwise distances and since its power demand for single flight is very small, its relative power reduction  $e_3$  is very large. Accordingly contrary statements apply for formation H. For the whole formations it turns out that in both cases the inhomogeneity leads to a reduction of the total flight power reduction E.

Usually the formation flight is performed by birds of a single species. Nevertheless there may exist differences in size between adult and juvenile birds. The present investigations indicate that in this case the larger birds favour the smaller ones. Occasionally flight formations of birds of different species are observed; see e.g. R. Berndt, D. Hummel [15] and D. Hummel [12]. In this case for instance medium-sized birds should welcome large birds in their formation whereas small birds should be cast off. This behaviour of the medium-sized birds should take its rise from those positions located downstream of the larger or smaller birds since the benefits or disadvantages are most intensive there. However, it must be borne in mind that also non-aerodynamic factors effect the occurrence of mixed flight formations.

**Fig.6:** Distribution of flight power reduction in  $45^\circ$  swept V-formations of  $n = 7$  wings. Variation of the span of one wing.  
 $[\Delta\eta = 0, B_1 = 1 (\mu \neq 3), A_1 = 6$   
 $\text{and } c_{L\mu} = 1^\mu (\mu = 1, 2, \dots, 7)^\mu].$

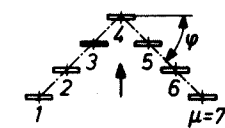
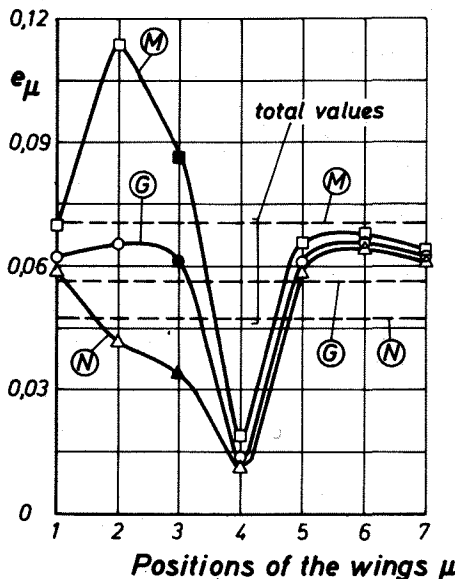


**Fig.7:** Distribution of flight power reduction in  $45^\circ$  swept V-formations of  $n = 7$  wings. Variation of the aspect ratio of one wing. [ $\Delta\eta = 0$ ,  $A_\mu = 6$  ( $\mu \neq 3$ ),  $c_{L\mu} = 1$  ( $\mu = 1, 2, \dots, 7$ )].

Similar results have been obtained for varying aspect ratio  $A$  in a formation. Fig. 7 shows again the  $\mu$  distribution of local flight power reduction  $e_\mu$  in the symmetrical  $45^\circ$  swept formation of  $n = 7$  equal wings with an aspect ratio  $A_\mu = 6$  ( $\mu = 1, 2, \dots, n$ ; formation G). The  $\mu$  wing  $\mu = 3$  is now replaced by a wing of twice aspect ratio  $A_\mu = 12$  (formation K) and of half aspect ratio  $A_\mu = 3$  (formation L). Since the wing spans and the spanwise distances are unchanged the interference between the wings  $\mu \neq 3$  is the same as in formation G. According to equ. (3) the high aspect ratio wing  $\mu = 3$  (formation K) has a smaller circulation, which leads to a decrease of the relative flight power reduction for all positions  $\mu \neq 3$  as compared with formation G. The high aspect ratio wing itself has a large relative flight power reduction due to its low flight power demand in single flight. Accordingly contrary statements apply for formation L. The inhomogeneity of a formation with respect to the aspect ratio can lead to larger or lower values of the total flight power reduction  $E$ . Details are discussed by D. Hummel [12].

The effect of varying wing loading is studied in Fig. 8. Starting-point is again formation G of  $n = 7$  equal wings which fly

**Fig.8:** Distribution of flight power reduction in  $45^\circ$  swept V-formations of  $n = 7$  wings. Variation of the loading of one wing. [ $\Delta\eta = 0$ ,  $A_\mu = 6$  ( $\mu = 1, 2, \dots, 7$ ),  $c_{L\mu} = 0.3^\mu$  ( $\mu \neq 3$ )].



**Formation M**  
 $(G/F)_3 = 2(G/F)_{\mu \neq 3}$

**Formation G**  
 $G/F = \text{const.}$

**Formation N**  
 $(G/F)_3 = (1/2)(G/F)_{\mu \neq 3}$

now at a lift coefficient  $c_{L\mu} = 0.3$  ( $\mu = 1, 2, \dots, n$ ). The wing  $\mu = 3$  then replaced by a twice loaded wing ( $c_{L3} = 0.6$ , formation M) and by a half loaded wing ( $c_{L3} = 0.15$ , formation N). The interference between the wings  $\mu \neq 3$  is unchanged. The higher loaded wing induces more upwash and therefore the relative flight power reduction increases for all positions  $\mu \neq 3$  as compared with formation G. The benefit is extremely large for the wing  $\mu = 2$  flying downstream of the heavier wing  $\mu = 3$ . Whether the relative flight power reduction of the varied wing  $\mu = 3$  increases or decreases depends on the lift coefficients. In the present case, the power reduction in position  $\mu = 3$  is also increased. This is due to fact, that the level of lift coefficients  $c_{L\mu} = 0.3$  ( $\mu \neq 3$ ) in the formation  $L_\mu$  has been chosen very low and wing  $\mu = 3$  flies at  $c_{L3} = 0.6$  at a better drag ratio  $(c_{D,ell} / c_{D,z})$ . Accordingly opposite statements apply to formation N. For the present examples it is evident from the local values of the relative power reduction that the total relative power reduction  $E$  increases for formation M and decreases for formation N as compared with formation G.

In flight formations of birds of a single species there exist sometimes considerable differences in weight, especially between males and females. The present investigations show, that the heavier individuals favour the light ones. Vice versa a weak individual is automati-

cally favoured by all other individuals of the formation. This means that in flight formations weak individuals have a better chance for survival. In mixed formations of birds of different species medium-weight birds should welcome heavier birds in their formation whereas lighter birds should be cast off. This behaviour should again take its rise from those individuals flying downstream of the heavier or lighter birds.

### 3.4 Outlook

For small spanwise distances the relative power reduction in flight formations reaches considerable values. Moreover, the relative flight power reduction can be further increased by a reduction of the flight speed of the formation below the speed in single flight. This reduction is due to the fact that taking into account the reduction of induced drag the optimum drag ratios for maximum range flight or minimum power demand flight lie at larger lift coefficients or at lower speeds.

In order to reduce the fuel consumption of aircraft and thus to reduce the direct operation costs (DOC) it would be desirable to make good use of this interference effect. For the symmetrical wings of aircraft in formation flight an unsymmetrical spanwise distribution of circulation turns out. The corresponding rolling moment can

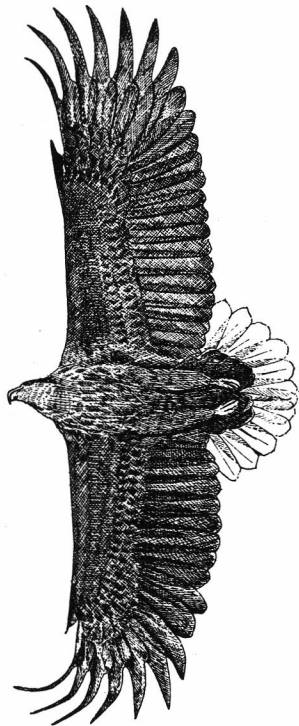


Fig.9: Soaring White-Tailed Eagle (*Haliaeetus albicilla*) from below (after a photo by E. Hosking).

be compensated by a proper deflection of the ailerons. Calculations after the lifting surface theory of E. Truckenbrodt [13] showed that the needed flap deflection angles are very small and that a final spanwise distribution of circulation can be achieved, which is very close to the elliptic one and which leads to a drag reduction as compared with single flight. This means that the positive interference effects might also be utilized by aircraft. A study of the possibilities based on lifting surface theory is in progress.

Formation flights of sailplanes at very low spanwise distances have been carried out at the Technical University of Braunschweig by the Akademische Fliegergruppe Braunschweig (AKAFLIEG). It turned out that in swept formations of two equal sailplanes at small spanwise distance a smaller sinking velocity is obtained at the sailplane in the rearward position. This result must be regarded as a verification of the theoretical considerations.

## 4. Slotted wing-tips in birds

### 4.1 Background

In gliding flight a large number of birds such as for instance Hawks (Accipitridae), Storks (Ciconiidae), Cranes (Gruidae) and Pheasants (Phasianidae) show distinctly splitted wing-tips. The wings of the birds of these families have a relatively small aspect ratio. On the other hand Swallows (Hirundinidae), Swifts (Apodidae) and all seabirds such as for instance Albatrosses (Diomedidae) and Gulls (Laridae) have high aspect ratio wings with pointed wing-tips which are really not splitted.

The geometry of slotted wing-tips has been comprehensively described by R.R. Graham [16]. Recent quantitative additions are due to H. Oehme [17]. According to Fig. 9 the wing consists of a closed inner part which is equipped in the tip region by some winglets. The wing planform is therefore split up. The wing is closed up to the proximal part of the outer primaries where the webs overlap. The winglets themselves consist of the emarginated distal part of the outer primaries. The winglets are bent upwards by aerodynamic forces in such a way that the winglet at the leading edge is located in the uppermost position whereas the winglet at the trailing edge is the lowest one. This means that the winglets are staggered in height. Some of the winglets, especially those near the wing leading edge, are twisted in such a way that their geometrical angle of incidence increases in distal direction. These details have been analyzed recently by D. Hummel [18].

On the aerodynamic meaning of slotted wing-tips in the biological sciences exist some suppositions based on comparisons with man made wings. A first assumption is that slotted wing-tips are a high lift



device since they look like a multi airfoil system. But this explanation might not be correct for two main reasons: Firstly, the distance between the winglets is too large, especially in their tip region, in order to obtain a multi airfoil effect. Secondly, high lift devices should be located at the proximal part of the wing rather than in the low loaded distal parts where the local lift coefficient tends to zero at the wing tip. A second assumption is that slotted wing-tips might be a device to reduce drag. This explanation has been proposed originally by R.R. Graham [16] and has been supported later by H. Hertel [19], H. Oehme [17] and others. H. Hertel [19] treats the problem only as a matter of course, whereas H. Oehme [17] performs calculations of the drag reduction which are in contradiction to well known principles of aerodynamic theory.

In the aeronautical sciences it is well known since M.M. Munk [14] that the induced drag of plane wings cannot be reduced below the minimum value

$$D_{im} = \frac{2 L^2}{\pi q V^2 b^2} \quad (16)$$

(L = lift,  $q = \rho V^2 / 2$  = dynamic pressure, b = span) which occurs for elliptic spanwise distribution of circulation. B.G. Newman [20] has shown anew that plane lifting systems of tandem wings cannot lead to any reduction of induced drag as proposed recently by H. Oehme [17]. The lifting system has to be a non-planar one to obtain an effect of drag reduction. C.D. Cone Jr. [21,22] and G. Löbert [23] investigated the possible induced drag reduction for various non-planar lifting systems. Since then a large number of papers appeared on the drag reduction of wings of aircraft by a supplementary winglet equipment, see for instance G. Löbert [23], H. Zimmer [24], B. Ewald [25], R.T. Whitcomb et al. [26], K.K. Ishimitsu [27]. The principles of drag reduction by non-planar lifting systems are now available to be applied to the corresponding problem of bird flight. Following here some theoretical and experimental results for bird configurations with winglets will be presented.

#### 4.2 Aerodynamic theory

The aerodynamic characteristics of wings with slotted wing-tips have been investigated by means of aerodynamic theory. The calculations have been carried out using a numerical panel method such as for instance described by S.R. Ahmed [28]. The wing thickness has not been taken into account. This means that actually a vortex lattice method with straight free vortices parallel to the free stream direction has been used. The induced drag has been calculated for given distribution of circulation in the lifting surface by integration along the trace of the vortex sheets in the Trefftz-plane. Subsequently some results are presented.

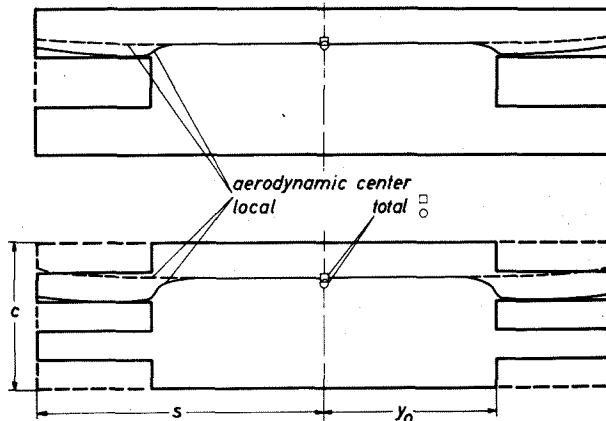


Fig.10: Position of the local and total aerodynamic centre for wings without and with slotted wing-tips ( $A = 2s/c = 4$ ,  $\eta_0 = y_0/s = 0.6$ ,  $n = 2$ ).

Fig. 10 shows the planform effect. For a rectangular wing of aspect ratio  $A = 4$  the spanwise slope of the local aerodynamic center as well as the total aerodynamic center are indicated. The basic wing has been slitted at a slitting ratio  $\eta_0 = y_0/s = 0.6$  in two different ways into two winglets at each wing-tip. It turns out that the local aerodynamic center is shifted rearwards in the tip region of the wing. Accordingly also the total aerodynamic center is shifted rearwards by slitting the wing-tips. This leads to a considerable increase of longitudinal stability.

A typical example for the drag characteristics is shown in Fig. 11. As a basis a rectangular wing of aspect ratio  $A = 4$  is used, the wing-tips of which are slitted at a slitting-ratio  $\eta_0 = y_0/s$  into  $n$  winglets of equal chord-length. First of all it turns out that for all plane configurations ( $\delta_{l,t} = 0$ ) with arbitrary values for  $\eta_0$  and  $n$  including the limiting cases of  $n$  tandem winglets ( $\eta_0 = 0$ ) the induced drag  $D_i$  is larger than the minimum value for elliptic spanwise distribution of circulation according to equ. (16). The ratio  $D_i/D_{im}$  is close to unity and depends slightly on the number of winglets and on the slitting ratio  $\eta_0$ . Fig. 11 contains also results for non-planar lifting systems. In these calculations the leading winglet was set at a stagger angle of  $\delta_{l,t} = +20^\circ$  and the trailing winglet at  $\delta_{l,t} = -20^\circ$ . For  $n > 2$  winglets the angle

$$\Delta\delta = \delta_{l,t} - \delta_{t,t} = 40^\circ$$

has been divided into equal parts  $\Delta\delta/(n-1)$ . For the non-planar wings a drag reduction results which increases with decreasing slitting ratio  $\eta_0$  and with increasing number of winglets  $n$ . If the stagger angle  $\delta_{l,t}$  is increased the induced drag reaches a minimum, which depends also on  $\eta_0$  and  $n$ . For  $n = 3$  and  $\eta_0 = 0.7$  its value is  $D_i/D_{im} = 0.86$  at

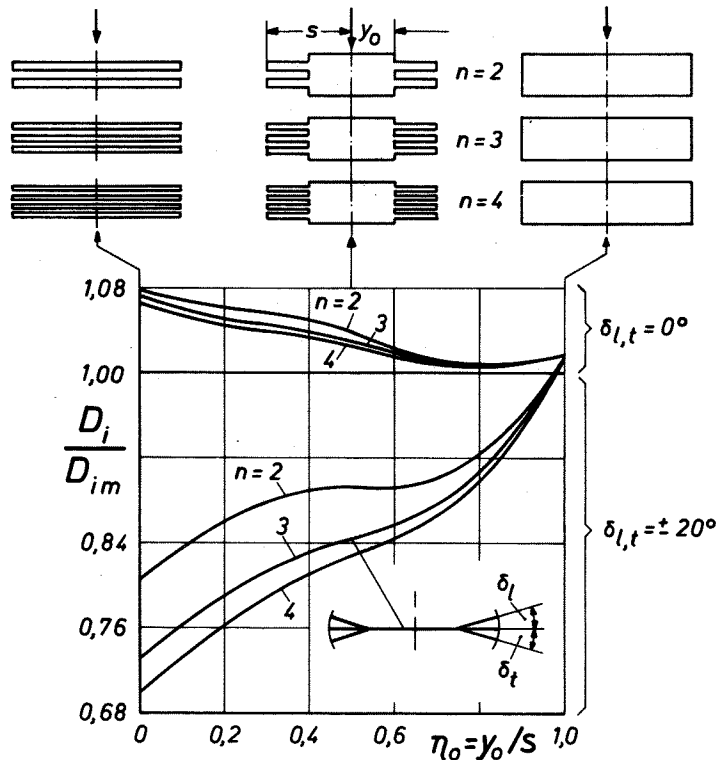


Fig.11: Theoretical induced drag characteristics of planar and non-planar wings with slotted wing-tips ( $A = 2s/c = 4$ ). Planar:  $\delta_l = \delta_t = 0$ . Non-planar:  $\delta_{l,t} = \pm 20^\circ$ .

$\delta_{l,t} = \pm 35^\circ$ . These results indicate that the possible drag reductions are moderate. Moreover, it has to be borne in mind, that the friction drag is increased in the slotted region of the wing since the number of leading-edges has increased there. This means, that the net drag reduction is really moderate.

Another result of these calculations, which is not shown here, are very large local lift coefficients which occur in

the root section of the winglets located close to the leading-edge. Since flow separations at these winglets would reduce the benefit once more, the winglets have to be twisted in such a way that their geometrical angle of incidence decreases in proximal direction. These details are observed in bird wings and have been described in section 4.1; see also D. Hummel [18]. It turns out that the drag reduction may only be achieved by a proper arrangement of the winglets.

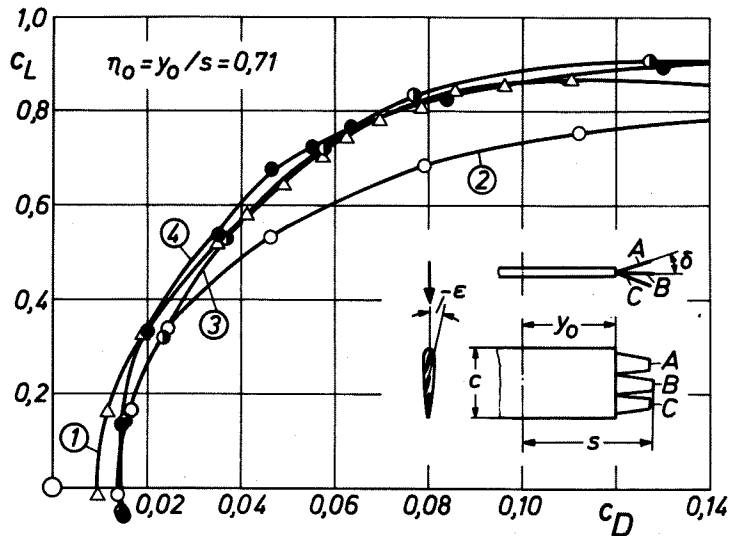


Fig.12: Experimental lift-drag characteristics of planar and non-planar wings with slotted wing-tips (Notations 1 to 4 see Fig. 13).

① Rectangular wing,

△ A=4

② Slotted wing-tips

○  $\delta_{A,B,C} = 0^\circ/0^\circ/0^\circ$

$\epsilon_{A,B,C} = 0^\circ/0^\circ/0^\circ$

③ Slotted wing-tips

●  $\delta_{A,B,C} = 0^\circ/0^\circ/0^\circ$

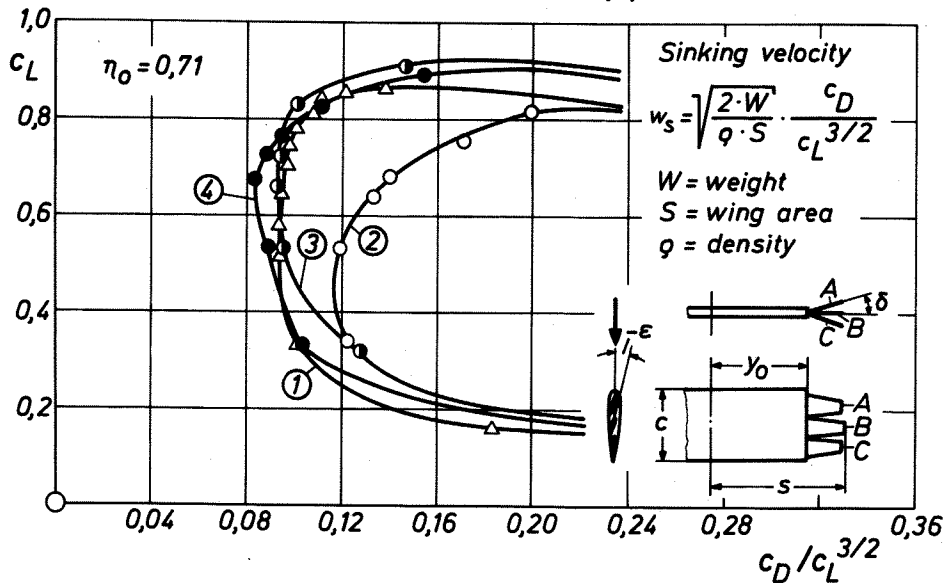
$\epsilon_{A,B,C} = -10^\circ/-5^\circ/0^\circ$

④ Slotted wing-tips

●  $\delta_{A,B,C} = 20^\circ/0^\circ/-20^\circ$

$\epsilon_{A,B,C} = -10^\circ/-5^\circ/0^\circ$

Fig. 13: Experimental sinking velocity characteristics of planar and non-planar wings with slotted wing-tips



#### 4.3 Experimental investigations

The measurements have been carried out in the 1,3 m wind-tunnel of the Institut für Strömungsmechanik at the Technische Universität Braunschweig. The model consisted of an inner rectangular wing which could be equipped at each wing-tip with up to 3 winglets. The slitting ratio was  $\eta_0 = 0,71$  and the aspect ratio of the unslitted outer rectangular wing was  $A=4$ . The main wing and the winglets had symmetrical airfoil sections NACA 0015. The winglets were pointed from  $2c/7$  at their root to  $c/7$  at their tip ( $c$  = chord of the main wing) and could be arranged at certain stagger angles  $\delta$  and at twisting angles  $\epsilon$  which were constant along their span.

Fig. 12 shows the polar diagramm for different configurations. The unslitted rectangular wing (curve 1) has the lowest drag coefficients. Slitting of the wing-tips (curves 2,3,4) increases the drag coefficient at zero lift due to increased friction drag. Slitting the wing-tips into a plane untwisted system of winglets (curve 2) leads to a considerable increase of drag and to a reduction of maximum lift coefficient due to flow separations at the winglets. These can be avoided by a proper distribution of the twisting angles  $\epsilon$  at the winglets. The best plane configuration (curve 3) has again about the same maximum lift coefficient as the rectangular wing. If the winglets are staggered in height (curve 4) the drag is reduced at the same

amount as indicated by the theoretical predictions.

In order to show the effect of drag reduction on the velocity of sinking in the case of a gliding flight the experimental results are drawn again in Fig. 13. The quantity  $c_D/c_L^{3/2}$  which is proportional to the sinking velocity

$$w_s = \sqrt{\frac{2W}{\rho S}} \frac{c_D}{c_L^{3/2}}$$

is plotted versus the lift coefficient. It turns out that the relatively small drag reduction by staggering the winglets in height causes a considerable decrease of the minimum sinking velocity.

#### 4.4 Discussion

Apart from the planform effect of increased longitudinal stability which might be used by birds for control purposes the main effect of slotted wing-tips is the moderate drag reduction due to staggering the winglets in height. The same effect could also have been achieved by a slight increase of wing span. If the way of life of a bird allowed a slightly larger wing span to increase its aerodynamic efficiency the evolution might have developed the wings in this direction. Examples are Swallows, Swifts and all seabirds which have really unslitted wing-tips. If on the other hand the way of life of a bird did not allow to exceed a certain wing span slitting of the wing-

tips and staggering the winglets in height is a device to increase the aerodynamic efficiency at this limiting condition. An example of this kind are the Pheasants. Their span is so strongly limited that they have developed the largest number of winglets of all birds in order to achieve sufficient flight performance.

A large number of birds such as Storks, Cranes, Hawks and Vultures use their wings in two different configurations: In gliding flight the wing span is reduced and the wing-tips are closed. In this flight condition the lift coefficient is low and the main portion of drag is friction drag which is kept small by a small surface. There is not need to reduce induced drag and, therefore, the wing-tips are closed. In soaring flight these birds circle in thermals at low speed. In this flight condition the lift coefficient is high and the main portion of drag is induced drag. In order to keep this portion of drag small and thus to achieve a small sinking velocity the wing span has to be as large as possible and the wing-tips must be slotted.

According to D. Hummel [18] the basic results reported here might also be important for certain periods of the flapping cycle of an even larger number of bird species. Almost all songbirds are equipped with emarginated primaries which are extremely spread during the downstroke. This indicates that slotted wing-tips are also used in this flight condition to produce a maximum of thrust from a wing of limited span.

## 5. Flapping flight of birds

### 5.1 Background

The thrust force necessary for the forward motion of a bird is generated by beating and pitching oscillations of the wings. This kind of thrust production has been studied and described by means of simple examples in the papers of D. Küchemann and E.v.Holst [29,30]. The sequence of wing motions of a bird is qualitatively well known from many slow motion films, but quantitative investigations of the wing geometry during the flapping cycle did not exist till a few years ago. For the lack of precise data E.v.Holst [31] analysed artificial birds the wing motions of which were realised in experiments by flying models with simple mechanisms. A few years ago D. Bilo [32,33] succeeded in the quantitative experimental determination of the complete flapping-cycle kinematics of a bird. The measurements have been carried out using high-frequency stereo-photogrammetry on a House-sparrow (*Passer domesticus* L.) flying free in a wind-tunnel. These experiments are till now unique and they are concerned with the most complicated flapping motion in forward flight of birds. There exist other flapping modes in birds, e.g. that described by H. Oehme [34] for a medium-sized bird, which have not yet been in-

vestigated quantitatively.

The flapping flight of birds will only be understood in details, if all the different modes have been determined quantitatively and if the corresponding aerodynamics have been calculated over the whole flapping cycle for all these modes. A first attempt in this direction has been carried out by D. Hummel and W. Mölbenstädt [35] for the experimental data of D. Bilo [32]. Following here the analysis is summarized and some results are given.

### 5.2 Aerodynamic analysis

Although the geometry has been determined by D. Bilo [32,33] for the whole flapping cycle the details are completely described only for one moment in the middle of the downstroke when both wings were nearly in a horizontal position.

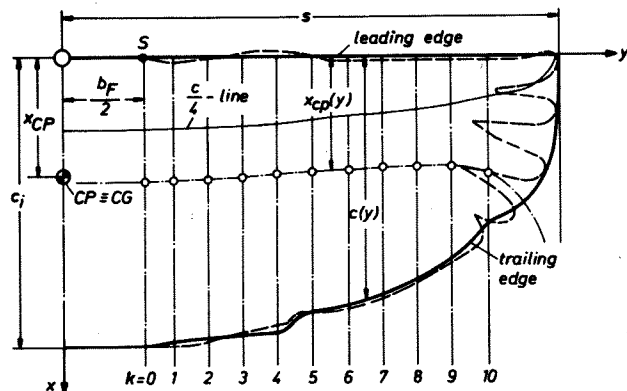


Fig.14: Wing planform of a Sparrow during the downstroke

--- measured by D. Bilo [32]  
 — replaced for calculations

For this special moment the wing planform is drawn in Fig. 14. In addition the dorsal wing contours are available for 11 particular sections, indicated in this figure. In order to carry out aerodynamic calculations it is necessary to determine the mean camber lines of the profiles in these particular sections. This problem has been solved approximately using the thickness distribution of a House-sparrow wing given by H. Oehme [36]. The resulting airfoils are shown in Fig. 15.

According to D. Bilo [32] the flapping frequency is  $f = 21.8$  [1/sec]. For a beating part of the semispan (semispan  $s$  minus half width of the body  $b_p/2$ ,  $s' = s - b_p/2$ , see Fig. 14) of  $s' = 97$  [mm] and a flight speed of  $V = 9$  [m/s] the reduced frequency according to D. Küchemann and E.v.Holst [29,30] is  $\omega = 0.23$ . For this value instationary effects may be present. Because of the lack of geometrical details for the whole flapping cycle a theoretical treatment including instationary effects is not possible. Therefore, these effects could not be taken into consideration, but

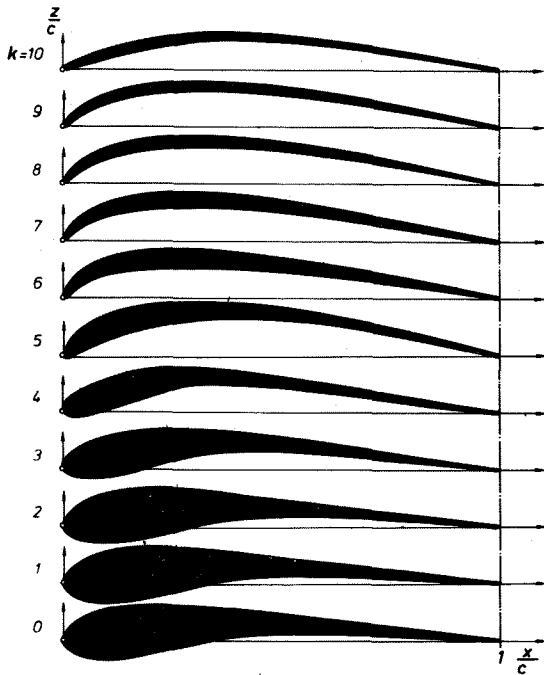


Fig.15: Profiles of a Sparrow wing during downstroke after D. Bilo [32] and H. Oehme [36].

it is thought that the present results are a reasonable approximation for that part of the forces acting in phase with the motion.

For the special moment during the downstroke for which the geometrical details are available from the measurements of D. Bilo [32] the aerodynamic forces and moments have been calculated by a quasistationary theory. The momentary position of the wings as well as the local velocity vectors resulting from the momentary forward-beating- and pitching-motion have been taken into account. The calculations have been carried out using the lifting surface theory after E. Truckenbrodt [13]. Details of the analysis may be taken from D. Hummel and W. Möllenstädt [35].

### 5.3 Results and discussion

As a result of the calculations the spanwise distributions of the local vertical and horizontal force coefficients are shown in Fig. 16. It turns out that all parts of the wing-span are concerned in producing vertical forces. The corresponding lift coefficients lie in the same order of magnitude as the vertical force coefficients. With respect to the low Reynolds number the resulting values must be regarded as large. However, it is remarkable that the bird moves its wings in such a way that the highest lift coefficients occur about the middle of the semispan near the wrist. In this region the birds are equipped with a small winglet called *Alula spuria* which might be used as an auxiliary device to produce

large lift coefficients. The distribution of the thrust coefficient shows that the body of the bird as well as the proximal parts of the wing cause small negative values of thrust (this means drag) while large wing portions in the distal region produce positive thrust. Instead of pitching-moment characteristics in Fig. 14 the local centres of pressure as well as the total centre of pressure have been drawn. Although the centre of gravity of the bird measured by D. Bilo [32] is not known it can be recognized from the position in Fig. 14 that the theoretical result seems to be in a reasonable order of magnitude.

The results presented here are concerned with one special moment during the downstroke. It would be most interesting to carry out calculations of this kind for other moments too, on order to be able to pursue the mechanism of producing vertical and horizontal forces through the whole flapping cycle. If the wing positions and motions would be available for other moments the calculations could also be extended in principle to include instationary effects. But it has to be borne in mind that as a result of the measurements of D. Bilo [32,33] the wings perform a very complicated threedimensional motion: The downstroke is combined with a forwardstroke. At the beginning and at the end of this part of the flapping cycle the positive and negative dihedral is considerably. This means that a nonplanar lifting system has to be treated. At the beginning of the upstroke the primaries are folded and at the end of the upstroke they are unfolded. This means that the aspect ratio of the wings varies considerably during the flapping cycle. Probably there exist also not so much complicated wing motions in birds. Therefore, further measurements after the method of D. Bilo [32] are needed for

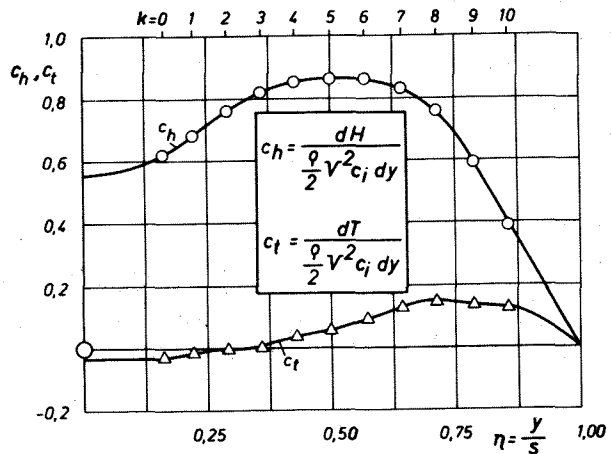


Fig.16: Spanwise distributions of the local vertical and horizontal force coefficients for a Sparrow wing during downstroke as calculated after lifting surface theory.

other birds. Moreover the theoretical methods should be extended to include instationary effects as well as the very complicated motions which occur in the flight of birds.

#### 6. Concluding remarks

It has been the purpose of this survey to show some aerodynamic problems which arise if the flight of birds is analyzed in detail. For all three selected problems, presented here, it turns out that they cannot be solved directly by measurements on birds. The aerodynamic forces acting on birds in soaring or flapping flight cannot be determined by wind-tunnel experiments. The only approach which could lead to a satisfactory solution of the problems and to a deeper understanding of the mechanisms of bird flight seems to be a quantitative experimental investigation of the positions and motions of birds in flight and a subsequent analysis by means of aerodynamic theory. All three problems described here have been treated in this way. The present results indicate that this approach is very useful and leads to new insights into the mechanisms of flight in nature.

Another conclusion which can be drawn from the present approach is the recognition that the problems of bird flight cannot be solved either by the biological sciences or by the aeronautical sciences themselves. The quantitative determination of the positions and motions of birds in different flight modes are problems of zoology whereas calculations of the corresponding aerodynamic characteristics are problems of theoretical aerodynamics. Therefore future progress in this field can only be expected from a close connection and cooperation between both scientific disciplines.

As a basis for aerodynamic investigations in the biological sciences the qualitative description of the flight of birds has to be replaced by quantitative investigations. A large number of problems are still unsolved. The method of D. Bilo [32,33] should be applied to all flight-modes of birds and the details should be described for the whole flapping cycle. Similar investigations are also needed for the stationary gliding and soaring flight of birds in order to obtain quantitatively the profiles as well as the bending and twisting of the primaries in the vicinity of the wing-tips.

In the aeronautical sciences a large number of theoretical methods are available for the calculation of the stationary and instationary aerodynamic characteristics of planar lifting systems of fixed planform shape. In most cases a direct application of these methods to the problems of bird flight is not possible since the geometry is much more complicated and variable in birds. Therefore the methods

have to be extended and improved for application in bird flight problems.

In both scientific disciplines interesting problems of bird flight are still unsolved. Efforts on both sides and a close cooperation will lead to further progress in this field.

#### 7. Summary

In the last years remarkable progress has been achieved by biologists in the quantitative determination of positions and motions of birds in flight. The aeronautical sciences are now challenged to contribute to the solution of the corresponding flow problems. Three of them are analysed here in some detail, namely

- (i) the formation flight of birds. It is shown by aerodynamic calculations that a power reduction occurs in such formations which depends on the number of birds, on their spanwise distance, on the shape of the formation as well as on the distribution of weight, span and aspect ratio of the wings in the formation.
- (ii) the slotted wing-tips. It is shown by theory and experiment that this device increases static stability and reduces induced drag. The benefit is moderate and can only be realised by a proper arrangement of the winglets.
- (iii) the flapping flight of birds. Some calculations have been carried out for measured wing positions and motions during the downstroke period. Spanwise distributions of vertical force and thrust are given.

The results, presented here indicate that the application of the methods of aerodynamics to this kind of problems is very useful and leads to a better understanding of principles and details of flight in nature.

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